

Research note

Limits on lateral density and velocity variations in the Earth's outer core

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Summary. An approximate analysis is given for the likely fractional lateral density variations ($\delta\rho/\rho$) in the outer core, caused by large scale-length fluid dynamical processes. It is first shown that fractional density and fractional seismic velocity variations are probably comparable, so that fluid dynamic arguments have relevance to seismic data. In regions of nearly neutral stability in the outer core, an analysis of convective vigour indicates an upper bound of $\delta\rho/\rho \lesssim 10^{-8}$. If the outer core possesses one or more layers of strong static stability then stationary contributions to $\delta\rho$ can be larger, if they are associated with axisymmetric ($m = 0$) harmonics, because of stabilizing zonal winds. Baroclinic instabilities nevertheless limit $\delta\rho/\rho \lesssim 10^{-6}$ but may not exist if the static stability is sufficiently large. Shear instabilities always limit $\delta\rho/\rho \lesssim 10^{-4}$. Magnetic field effects suggest comparable or more stringent upper bounds. It is concluded that scientists undertaking analysis of the Earth's geoid or seismic travel times or normal modes can safely assume that there are negligible lateral variations in the outer core.

Key words: core, density, fluid velocity, lateral inhomogeneity

1 Introduction

The increasing sophistication of analyses of geoid and seismic data prompts consideration of the question: Can we ignore lateral variation in the outer core? Many fluid dynamicists who have an understanding of Earth core properties consider this to be a 'trivial' question with an unequivocal answer: Yes. Unfortunately, this answer is based, at least in part, on certain common preconceptions and assumptions about how the core operates. Furthermore, the quantitative basis for this conclusion has never been enunciated clearly in a single place. The purpose of this note is to clarify and justify the expectation that lateral variations are negligible. In fact, the question is not an entirely trivial one and involves some interesting fluid dynamical issues.

We must first define what is meant by 'lateral variation'. Here, we mean any variation of a relevant physical property (density, temperature, seismic velocity) on a surface of constant

gravitational potential, measured in a rigidly rotating frame fixed to the Earth's surface angular velocity. These variations could arise because of core dynamics (convection, waves, other large-scale circulation including differential rotation) or could be excited from above because of variations in the angular velocity of the frame of reference (the mantle). We focus on the former since the latter appears to be small, both energetically and dynamically. For example, precessionally induced flows, probably the strongest of the externally imposed flows, appear to be unimportant for the geodynamo (Rochester *et al.* 1975; Loper 1975) and will therefore be even less important to lateral variation than the examples discussed below.

Before embarking on any detailed quantitative analysis, it is useful to review why the core is very different from three better understood geophysical fluid dynamical systems in which significant lateral variation can occur: the mantle, the ocean, and the atmosphere. The core differs from the mantle because it has an immensely different rheology. It differs from the ocean and atmosphere because it is much less strongly forced (i.e. geophysical heat flow \ll solar energy flux). Let us discuss each in turn.

Post-glacial rebound, geoid, and convection studies suggest a mantle viscosity of 10^{21} – 10^{22} Poise (see, for example, Turcotte & Schubert 1982). In this highly viscous regime, significant redistribution of heat or composition can only occur if the lateral density anomalies are substantial. For example, a 'blob' of material 1 per cent lighter than ambient conditions rises by Stokes flow and has an ascent time $\sim 10^9$ yr if it has a physical dimension ('wavelength') of ~ 100 km. Density variations of roughly this order are plausible for mantle convection (associated with lateral temperature variations of up to $\sim 10^3$ K) and, as argued below, fractional seismic velocity variations of comparable magnitude to the fractional density variations are to be expected. These appear to have been detected in the mantle (Hager *et al.* 1985).

The viscosity of the core is not well known precisely because (unlike the mantle) it is dynamically unimportant. At the top of the core, nutation studies limit the kinematic viscosity $\nu < 10^5$ cm² s⁻¹ (Toomre 1974; see also Rochester 1976). Although seismic constraints are more global, yet weaker (see summaries in Jacobs 1975; Lambeck 1980), theoretical estimates suggest values of $\sim 10^{-3}$ cm² s⁻¹, typical of liquid metals (Gans 1972; Gubbins 1976; Stevenson 1981). We proceed with the usual assumption that the core is 'inviscid', like the ocean and atmosphere, meaning that the amplitude of large scale motions (and their associated density variations) are not affected by the value of the viscosity. At the end of this note, possible relaxation of this assumption is considered. In the spirit of Kolmogorov scaling (e.g. Golitsyn 1979), one appropriate dimensionless number characterizing inviscid systems is $v_0^{-1}(F/\rho)^{1/3}$, where F is the heat (or energy) flux, ρ is the density and v_0 is some characteristic velocity (i.e. sound speed or free fall speed; the precise identification is not important here). In the next section, this number arises naturally in the determination of convective density fluctuations. In the atmosphere and ocean, F/ρ exceeds the value in the Earth's core by eight and four orders of magnitude, respectively. (For this estimate, an upper bound of 10^2 erg cm⁻² s⁻¹ is used for the core energy flux.) Since the value of v_0 is over an order of magnitude larger in the core than in the ocean or atmosphere, it follows that the core value of our dimensionless number is much smaller than for the ocean or atmosphere. The core is remarkably quiescent and it is correspondingly inappropriate to envisage 'storms' or other variations in the core of comparable vigour to those seen at the surface.

Since the fluid dynamical arguments developed below pertain to density rather than seismic velocity, the next section deals with the relationship between these quantities. We then analyse lateral variation in convecting layers. However, we do not want to assume that

the core is close to neutral stability (i.e. convective) everywhere, since it is not possible to exclude layers of large static stability either by seismic data or by analysis of the geodynamo (which could operate above or below this hypothesized static layer). Therefore, subsequent sections deal with lateral variability in the presence of static stability. We conclude with an assessment of the limitations of this analysis.

2 Correlation between density and seismic velocity

We wish to consider

$$A \equiv \left(\frac{\delta V_p}{V_p} \right)^2 / \left(\frac{\delta \rho}{\rho} \right) = \frac{d \ln V_p}{d \ln \rho}, \quad (1)$$

where V_p is the P -wave velocity. In a liquid, $V_p = (K_s/\rho)^{1/2}$, where K_s is the adiabatic bulk modulus. The derivative in the definition of A is along a 'path' defined by the particular dynamic process envisaged, as explained below. Adiabatic perturbations for the seismic waves are assumed, but the analysis does not depend crucially on which bulk modulus is used in V_p .

The value of A is needed to translate fluid dynamical estimates of $\delta \rho/\rho$ to estimates of expected lateral seismic heterogeneities. Clearly, a large A enhances the sensitivity of seismic techniques.

There are several possibilities:

(i) In purely thermal convection, a fluid element may have almost the same pressure but a significantly different temperature than a neighbouring element on the same equipotential:

$$\begin{aligned} A &= \frac{1}{2} \left[\left(\frac{\partial \ln K_s}{\partial \ln \rho} \right)_p - 1 \right] \\ &= \frac{1}{2} \left[\left(\frac{\partial \ln K_s}{\partial T} \right)_p \left(\frac{\partial T}{\partial \ln \rho} \right)_p - 1 \right] \\ &= \frac{1}{2} (\delta_s - 1), \end{aligned} \quad (2)$$

where δ_s is an 'anharmonicity parameter' and has a typical value ~ 5 in materials of geo-physical interest (Anderson & Suzuki 1983). This suggests $A \sim 2$.

(ii) In waves, we might be more concerned with

$$\begin{aligned} A &= \frac{1}{2} \left[\left(\frac{\partial \ln K_s}{\partial \ln \rho} \right)_s - 1 \right] \\ &= \frac{1}{2} \left[\left(\frac{\partial K}{\partial P} \right)_s - 1 \right] \\ &\sim 2 \end{aligned} \quad (3)$$

using seismically determined values of the parameters (see Jacobs 1975).

(iii) In compositional convection, two fluid elements on a given equipotential might have almost the same P , T but different composition. No simple result is possible here, and one

could imagine *special* cases where A is very large (just as it is possible to have alloys in which the coefficient of thermal expansion is 'anomalously' small). However, the systematics of elements and compounds (Birch 1968) indicate $A \sim -1$ is typical (note the change in sign).

(iv) The core could have constant composition but have suspended solids, with lateral variations in the suspension load. This is more difficult to assess and is the most plausible candidate for a large A , since one could imagine a liquid and a solid of significantly different sound speed yet almost identical density. Even so, large values of A seem unlikely in the core. For example, the P -wave velocities of solid and liquid iron differ by only ~ 5 per cent at the ϵ -liquid transition at $P \sim 2$ Mbar (Brown & McQueen 1980). Since the density difference between pure solid and pure liquid is ~ 2 per cent, we get $A \sim 2.5$.

We conclude that very large values of A are unlikely. Values of order 2 are most likely, but note that the *sign* of A is very uncertain.

3 Lateral variations in a convecting layer

The theory of core convection is not as well advanced as the theory of mantle convection, for three reasons: (i) core convection is dynamically more complex: rotation and magnetic field are important; (ii) core convection is likely to be turbulent (high Reynolds number flow); (iii) the energy source and buoyancy sources in the core are not well understood.

These uncertainties are not an excuse for postulating large density variations, but they do mean that estimates of the likely density variations have large (\sim factor of 10) uncertainties. This is sufficiently accurate for our purpose.

If thermal convection operates, then a simple 'mechanistic' argument, known as mixing length theory (e.g. Clayton 1968; Frazer 1973; Stevenson 1979) relates convective heat flux F , convective velocity v , and associated density variations $\delta\rho$ by

$$F \simeq 0.1 C_p \delta\rho v / \alpha$$

$$v \simeq (gl \delta\rho / \rho)^{1/2}, \quad (4)$$

where $C_p \simeq 10^7$ erg g^{-1} K^{-1} is the specific heat, $\alpha \simeq 10^{-5}$ K^{-1} is the coefficient of thermal expansion, $g \sim 10^3$ $cm s^{-2}$ is the gravitational acceleration, and $l \sim 10^8$ cm is the scale length of the convective motions. In turbulent convection with equidimensional eddies, $\delta\rho$ is both a measure of lateral variation on an equipotential and vertical variation relative to a mean state that is close to adiabatic.

Since $F \lesssim 10^2$ erg cm^{-2} s^{-1} (and possibly a lot less), it follows that $\delta\rho/\rho \lesssim 2 \times 10^{-10}$. This result is modified only slightly if compositional buoyancy is present (irrespective of the direction of the convective heat flow) because the energy available from redistribution of light material is not enormously larger than that from sensible heat. A bigger correction may arise from allowing for the effects of rotation and magnetic field (Stevenson 1979), perhaps increasing $\delta\rho/\rho$ to as much as 10^{-8} , but with a smaller associated wavelength. It is clear, however, that convective regions cannot tolerate significant $\delta\rho/\rho$ because too much heat flow (or too much redistribution of light material) would occur. It is possible to argue at great length about the level of inaccuracy in equation (4) and the corresponding uncertainty in $\delta\rho/\rho$ (see Stevenson 1979) but since the predicted value is *enormously* far away from detectability, these concerns are academic in the present context. Notice that for $\delta\rho/\rho \sim 10^{-3}$, the predicted convective velocity would be an astonishing 10^4 $cm s^{-1}$.

Of course, one cannot exclude very narrow jets or thin boundary layers with large $\delta\rho/\rho$ ($\sim 10^{-3}$, say) but these would not be detectable in seismic or geoid analyses. (They would be many orders of magnitude thinner than mantle plumes!)

4 Lateral variations in a stably stratified layer

This situation is more interesting and less straightforward. For definiteness, consider a layer of thickness D in which the density gradient is stable (i.e. more negative than that appropriate for self-compression of a constant composition, adiabatic fluid). We can define a Brünt–Vaisala frequency, N :

$$N^2 = -\frac{g}{\rho} \left(\frac{d\rho}{dr} - \frac{\rho}{K_s} \frac{dp}{dr} \right) \quad (5)$$

$$K_s \equiv \rho(\partial p / \partial \rho)_s,$$

where r is the radial distance, p is the pressure and K_s is the adiabatic bulk modulus. The frequency, N , is the oscillation frequency of a vertically displaced fluid element, in the absence of other dynamics. The layer could have ‘microstructure’ (e.g. a step-like density distribution of neutrally stable layers separated by strongly stable diffusive interfaces, as observed in double diffusive systems, including the Earth’s oceans; Turner 1973) and could have either thermal or compositional contributions to the stability. None of these complications affects the general principles developed below. An examination of existing seismic models for the outer core (Jacobs 1975; Lambeck 1980, p. 23; Dziewonski & Anderson 1981) and a comparison with equation of state data for possible core compositions (reviewed by Stevenson 1981) suggests that only $\lesssim 3$ per cent density deviation from an adiabatic, constant composition outer core is possible. (If the core were *isothermal* instead of adiabatic, this would produce $\lesssim 1$ per cent density variation.) However, this variation could occur on a much smaller scale length than the core radius, because seismic data cannot readily resolve rapid variations. We conservatively adopt

$$N \lesssim 5 \times 10^{-4} \left(\frac{10^3 \text{ km}}{D} \right)^{1/2} \text{ s}^{-1} \quad (6)$$

($D \lesssim 10^3 \text{ km}$) as an upper bound for the static stability. Of course, a small D would lead to correspondingly small effects in seismic or geoid data, because of the poor spatial resolution of these data.

If we impose a horizontal density gradient then surfaces of constant density are tilted relative to equipotential or constant pressure surfaces. This is directly analogous to undulations on a free water surface and will, in general, lead to propagating or standing waves. There is an important exception to this expectation in which stationary, purely meridional, density gradients ($m = 0$ harmonics) can be sustained because of a zonal shear (radial differential rotation of the core fluid) but we deal first with the tesseral harmonics ($m \neq 0$ in the usual spherical harmonic representation). For a horizontal scale length (wavelength) of L , the phase speed of the wave corresponding to these harmonics (relative to the ‘rest’ frame defined by the Earth’s mantle) is $\sim NL$. This neglects the effect of the Coriolis force, but since we are most interested in strong static stability $N \gtrsim \Omega$ (the planetary angular velocity $\sim 7 \times 10^{-5} \text{ s}^{-1}$), this is not a big error. The neglect of magnetic fields is even better justified. Typical fluid velocities are then $w \sim (g/N)(\delta\rho/\rho)$, where $\delta\rho$ is the magnitude of the lateral density variation. If Q is the quality factor for these waves (i.e. free decay would occur in $\sim Q$ periods) then these waves are dissipating energy at a rate $\sim N\rho w^2/2Q$ per unit volume. If the total dissipation is bounded above by $\sim 10^{20} \text{ erg s}^{-1}$ (an upper bound to energy out of the core), then

$$\frac{\delta\rho}{\rho} < 10^{-8} Q^{1/2} \left(\frac{N}{10^{-4} \text{ s}^{-1}} \right)^{1/2} \quad (7)$$

This is an extremely conservative upper bound because only a small fraction of the core energy flow is likely to be fed into these waves. Unless these waves are remarkably non-dissipative, the implied upper bound is very small. Although $Q \lesssim 10^4$ for seismic attenuation (including normal modes) in the core (Anderson 1980), there is no assurance that the same Q applies to these gravity waves. More importantly, these waves have rapid time-variability and would have a distinctive (non-stationary) effect in geoid and seismic data sets; we wish to focus here on stationary or slowly time-varying density variations.

Consider, instead, density variations which are spin-axisymmetric ($m = 0$). We neglect magnetic field effects for the moment. The curl of the Navier–Stokes equation (known as the vorticity equation) then admits a steady solution of the form

$$2\Omega \cos \theta \frac{\partial u}{\partial z} \simeq \frac{g}{L} \frac{\delta \rho}{\rho}, \quad (8)$$

where u is the zonal (east–west) wind, z is the local vertical direction, θ is the colatitude, and L is the (north–south) wavelength over which the density variation $\delta \rho$ occurs. This is the local Cartesian form of the ‘thermal wind equation’ (e.g. Pedlosky 1979, p. 42) well known to meteorologists. Physically, it represents the cancellation of two vorticity generating processes: baroclinicity (non-coincidence of the surfaces of constant pressure and constant density) and vortex tube stretching and tilting. (Non-fluid dynamicists should recall that an east–west wind produces a north–south Coriolis force, as required to balance the effect of a north–south density gradient.)

Equation (8) implies a typical zonal wind amplitude

$$u \simeq (10^4 \text{ cm s}^{-1}) (10^4 \delta \rho / \rho) \left(\frac{D}{10^3 \text{ km}} \right) \quad (9)$$

for $L \sim 10^9 \text{ cm}$ and $\theta \sim 45^\circ$. This could be very high compared with the differential rotation that is usually believed relevant to the core ($\sim 10^{-2} \text{ cm s}^{-1}$, typical of westward drift). However, it may be unstable to shear or baroclinic instabilities. We consider the latter first since they are most relevant unless the static stability is exceptionally strong. These baroclinic instabilities arise because there exist fluid motions which can release potential energy from the gravitational energy of the sloping density surfaces (see Pedlosky 1979, p. 451 onwards). The ‘local’ criterion for instability is

$$2\Omega \cos \theta L \gtrsim ND, \quad (10)$$

where L is the instability wavelength. In fact, L may be very large and a more accurate theory is needed (since θ is varying over the wavelength of the instability). For $N \sim 5 \times 10^{-4} (10^3 \text{ km}/D)^{1/2} \text{ s}^{-1}$ (our upper bound), $L \sim 5 \times 10^8 (D/10^3 \text{ km})^{1/2} \text{ cm}$ – uncomfortably close to the longest possible wavelength consistent with a sphere, if D is near its upper limit. This analysis is for a thin fluid layer, but a similar criterion applies in deep fluid shells (Ingersoll & Pollard 1982). If the instability is possible, then provided inequality (10) is satisfied by more than a small amount, the growth time of the instability is predicted by a simple model (Eady 1949) to be $\sim (\Omega L/g) (\rho/\delta \rho) \sim (10^2 \text{ s}) (\rho/\delta \rho)$. This would destroy the density variations on a very short time-scale. In fact, redistribution of material would be almost as rapid as the convective processes described in Section 2 above. To reduce the time scale to a dynamically acceptable value (e.g. 10 years) would require $\delta \rho/\rho < 10^{-6}$. Here, as in the convection analysis, the essential point is this: there is only a limited amount of energy that can be released through redistribution of buoyancy during the lifetime of the core. If a process exists which redistributes rapidly then it cannot persist; at best, it can be a transient effect.

It is possible, however, that the baroclinic instability is avoided. In these circumstances, a shear instability is still possible provided (Phillips 1951)

$$u \gtrsim \left(\frac{ND}{\Omega L} \right) ND. \quad (11)$$

For N near the upper limit, avoidance of this instability implies

$$\frac{\delta \rho}{\rho} \lesssim 10^{-4} \quad (12)$$

independent of the magnitude of D . As in the baroclinic case, the instability would, if initiated, redistribute material extremely rapidly. This is the least severe constraint on $\delta \rho$ that we obtain. Even if both baroclinic and shear instabilities are avoided, 'slow' meridional circulation persists and is likely to reduce $\delta \rho / \rho$ to smaller values than that implied by equation (12), unless there is some remarkably efficient process for re-establishing these density contrasts.

5 Magnetic field effects

For simplicity, the above bounds are derived neglecting the dynamics of the Lorentz force. A complete hydromagnetic generalization is difficult, but there are many indications that the geodynamo imposes comparable or stronger constraints on $\delta \rho / \rho$. Consider, for example, the zonal flow predicted by equation (9). If this were present, then a toroidal field $H_T \approx (uD/\lambda)H_p$ would be generated, for a pre-existing poloidal field H_p and magnetic diffusivity $\lambda \sim 10^4 \text{ cm}^2 \text{ s}^{-1}$. If we assume $H_p \sim 10$ Gauss (typical of the Earth's core) and require that the Ohmic dissipation associated with H_T is $\lesssim 10^{20} \text{ erg s}^{-1}$ then we find that

$$4 \pi^2 \lambda H_T^2 R^2 / D \lesssim 10^{20} \text{ erg s}^{-1}, \quad (13)$$

which implies that $H_T \lesssim (10^3 \text{ Gauss}) (D/10^3 \text{ km})^{1/2}$, or, equivalently, $u \lesssim (10^{-2} \text{ cm s}^{-1}) (10^3 \text{ km}/D)^{1/2}$, leading to $\delta \rho / \rho \lesssim 10^{-10} (10^3 \text{ km}/D)^{3/2}$. This is a rather similar estimate to the convective calculations (Section 2). It implies that very large zonal winds are intolerable unless that region is essentially field-free. As a corollary, regions containing substantial fields cannot contain large lateral density variations. Large zonal winds would also 'spin-axisymmetrize' the Earth's magnetic field (Stevenson 1982), inconsistent with observations. It is clear that the geodynamo argues against significant $\delta \rho / \rho$, irrespective of assumptions concerning its energy source and region of generation.

6 Limitations

The analysis presented here is approximate and non-rigorous; approximate because accuracy is not needed to demonstrate the point, and non-rigorous because rigour is an unattainable goal in this instance. (In geophysics, one should *never* avoid an issue or a computation simply because it cannot be done with rigour!) It is conceivable that the complicated, non-linear fluid dynamics of the core conspire to avoid the inequalities discussed here, through some neat balance of counteracting effects. The history of fluid dynamics suggests otherwise: it is very difficult to avoid instabilities. A conceivable limitation of our analysis lies in the possibility that the rheology of the deeper parts of the outer core is very different from that which characterizes the outermost part of the core (i.e. $\nu \lesssim 10^5 \text{ cm}^2 \text{ s}^{-1}$; Toomre 1974). Speculations concerning very high *bulk* viscosities exist (Anderson 1980; but see Stevenson

1983) but there is no current argument favouring $\gtrsim 10$ orders of magnitude viscosity increase within the outer core, as would be required to allow detectable buoyancy contrasts to persist. To be specific, the usual scaling arguments for thermal convection (based on boundary layer analysis, e.g. Turcotte & Schubert 1982) imply

$$F \cong \frac{k\Delta T}{L} \left(\frac{\text{Ra}}{\text{Ra}_{\text{cr}}} \right)^{1/3} \quad (14)$$

$$\text{Ra} \equiv \frac{g\alpha\Delta TL^3}{\nu k}$$

$$\text{Ra}_{\text{cr}} \simeq 10^3,$$

where F is the heat flux, k is the thermal conductivity, ΔT is the temperature difference driving the motions, Ra is the Rayleigh number, L is the depth of the fluid layer and K is the thermal diffusivity. If we suppose that ΔT is equal to its smallest detectable value of $\sim 10^2 \text{ K}$ (corresponding to fractional density variations $\sim 10^{-3}$ and comparable fractional velocity variations) and assume $F < 10^2 \text{ erg cm}^{-2} \text{ s}^{-1}$ then we derive a lower bound for ν . This yields

$$\nu \gtrsim 10^{14} \text{ cm}^2 \text{ s}^{-1} \quad (15)$$

(independent of L , but assuming $\alpha = 10^{-5} \text{ K}^{-1}$, $k = 10^5 \text{ erg cm}^{-1} \text{ K}^{-1} \text{ s}^{-1}$, $K = 10^{-2} \text{ cm}^2 \text{ s}^{-1}$). The dynamo could not operate in this region because the fluid velocities $v \sim K/L(\text{Ra}/\text{Ra}_{\text{cr}})^{1/3} \lesssim 10^{-5} \text{ cm s}^{-1}$ and the magnetic Reynolds number $\nu L/\lambda \lesssim 0.1$. This does not exclude this hypothesis, since the dynamo activity might be restricted to near the top of the core. Nevertheless, large viscosities as required by (15) must be regarded as very improbable since they appear to either violate seismic attenuation constraints (see Lambeck 1980) or require the existence of S -wave propagation in part of the outer core.

7 Concluding comments

We conclude that lateral variations in the outer core are either very small or have very rapid time variability (in which case they are unlikely to persist or be formed in the first place). If core-associated lateral effects exist then they are most probably at the core–mantle boundary or inner–outer core boundary. One could imagine, for example, ‘anti-lakes’: topographically confined pools of light fluid at the top of the outer core. Perhaps less plausibly, one could have analogous behaviour, including pronounced non-sphericity, at the bottom of the outer core. It is difficult to exclude inner core heterogeneities because we have no meaningful constraints on its rheology. It may not be ‘soft’ because the lateral temperature may be well below the melting point of pure iron. Nevertheless, these speculations should be regarded as acts of desperation. The outer core should not be considered as a likely source of lateral variability unless *all* alternative explanations have been exhaustively excluded.

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